Experiments with Aplusix in Four Countries

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The Aplusix system has been designed for helping students to learn algebra. Its capacity to tell the students whether their calculations are correct or not, to provide families of exercises of a chosen level, and to give scores after tests allows this system to be used in the regular functioning of the class. Its capacity to record the students' actions and to replay the students' work is a valuable feature for the researcher in mathematics education. In this paper, we describe four experiments conducted in four different countries with different goals: remediation piloted by researchers in Italy; remediation integrated in the regular functioning of classes in Brazil, collaborative learning in India, and learning and use during the entire school year in France.

1 INTRODUCTION

Aplusix (Nicaud, Bouhineau and Chaachoua., 2004; a demo version can be downloaded in English, French, Spanish and Portuguese from http://aplusix.imag.fr) is a computer program that helps students in learning algebra, in the field of numerical calculations, expansions, factorisations, equations, inequalities, and systems of equations, mainly for grades 7 to 10 (ages 12 to 16). Usually, Aplusix is installed on the server of the school and used by the students in the computer laboratory. An administration program allows teachers to create and manage classes and students' accounts. Aplusix contains 400 exercises grouped in families and structured in a *map of exercises*. They have random coefficients. Teachers also have the option of constructing their own lists of exercises through a special editor. The software records all the students' actions and allows the teacher to observe the students' work globally (through statistics) or in detail (with a replay system).

Aplusix is available in French, English, Portuguese, Italian, Spanish and Vietnamese and is being translated into other languages, in particular German, Japanese and Arabic. It has been marketed in France since 2005 and will be marketed in other countries in 2006.

One key feature of Aplusix is its advanced editor that displays and edits algebraic expressions in two dimensions. The editing facilities (input, delete, select, cut, copy, past, drag and drop) use the structure of the expressions, e.g. it is not possible to select $x+3$ from $2x+3$. With this component, students can do the calculations they want without any difficulties, either typing a new expression or modifying a copy of the previous one. Aplusix contains several modes. In the *training mode*, two fundamental feedbacks are provided to the student: the correctness of the calculation (see Figure 1) and the correct end of the exercise.

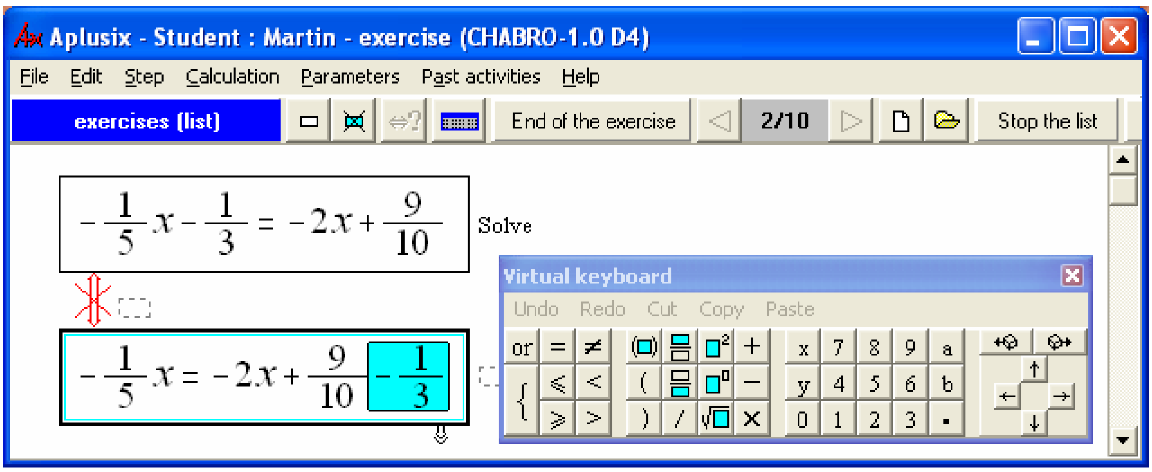

Figure 1 The *training mode* of Aplusix, also called *exercises activity*. The student makes his/her own calculations. When a step is correct, a black equivalence is drawn between the two boxes. When a step is incorrect, a red-crossed equivalence is drawn between the two boxes. In this figure, the student has moved a sub-expression (selection and drag & drop) producing a non-equivalent equation. He/She has to change the sign to obtain an equivalent equation.

In the *test mode*, no feedback is provided to the student. At the end of a test, the student gets a score and can enter the *self-correction mode* where the final states of the exercises are presented with the indication of the equivalences and non-equivalences. When this state is not correct, it can be modified by the student to obtain a correct one. The *observation mode* is a feature accessible to the student and the teacher. It allows one see the final state of solved exercises and to observe the resolution process, action by action.

The paper presents four experiments with the system prepared by researchers in mathematics education. The experiments involved a deep interaction between researchers and teachers to achieve a particular goal; or were mainly driven by the teacher to achieve a goal, or were used to evaluate the integration of Aplusix in the regular functioning of the class.

2 REMEDIAL PILOT INTERVENTION IN ITALY

The Italian study, conducted within the framework of the Kaleidoscope European network of excellence, is founded on the students' well-known difficulties in symbolic calculation (Freudenthal, 1983, Tall and Thomas, 1991 and Kieran, 1992). According to the Italian traditional school approach, at the 9th grade, a large part of the school time is devoted to symbolic calculation, which means that students are requested to memorise the formulas of the main products (second, third power of a binomial, difference of squares…) once they have been introduced to the notion of literal expression and to the main rules for expanding and factorising. Carrying out a symbolic calculation task quickly and accurately is one of the basic achievements of the first year of upper secondary school. Many students, for different reasons, encounter great difficulties in attaining this basic competence, which in itself seems to be only an application of rules, but which may become a great issue, both for the students and for the teachers. In order to analyse and, possibly, to give some suggestions to finding a solution to this didactical problem, we have focused our educational objective on specific aspects of students' difficulties in symbolic calculation.

We have formulated two related hypotheses. On the one hand, the natural complexity of memorising (Norman, 1988) leads to the *difficulty to memorise a specific formula*, even if its origin and its meaning have been well understood. On the other hand, an *aspect of the didactical contract* related to the algebraic calculation leads students to interpret the task only in terms of memorisation, without any other alternative strategies, when the required formula is not accessible.

Methodology

The research project has been conducted with the collaboration of a group of teachers. The interaction between teachers and researchers has been constant in every phase of the project: planning, experimentation in class and analysis of the collected data.

In order to test our specific hypotheses we have based the experiment on two goals: observing students' difficulties in algebraic calculation and finding a way to overcome these difficulties. Aplusix has been considered a useful tool regarding the two goals. In fact, on the one hand, it allows researchers to collect students' work and revise the sequence of solution, step by step, using the replay system, and on the other hand, it allows students to work in a well-structured environment.

The teaching sequence in three 9th grade classes is implemented as follows: a test on paper, a set of scenarios with the Aplusix environment and two final tests. The initial test is designed to reveal specific difficulties encountered by students in symbolic calculation and subsequently to work on and hopefully overcome them during the Aplusix sessions. The set of scenarios (which are made according to requirements identified in the different schools) is divided into different series, each aiming at a specific goal. Each series is made of a certain number of scenarios set with the *training mode* (with feedback), except for the last one which is set with the *test mode* (without feedback). The two final tests are performed in two different environments: the first one in Aplusix, the second one on paper. At the end of the test in Aplusix, students revise their work using the replay system. In this revision, the system indicates the errors as in the *training mode*, and students are asked to correct them in their notebook. The test on paper, identical to the test passed at the beginning, allows us to make comparisons between the students' production before and after the remedial intervention. Moreover, at the end of each session (tests inclusive) students are requested to report on what they think they have learnt, both in terms of formulas and in terms of strategies of solution, and comment on the use of Aplusix.

Results emerging from the data

The analysis of the data shows the effectiveness of the intervention not only in terms of a dramatic decrease in the numbers of errors, but also in terms of self confidence, self control and consciousness of the students about their own difficulties. The new relationship, between the machine and the student, seems to play a crucial role in overcoming the difficulties. In order to investigate how and why the microworld seems to be a useful tool, which identifies the students' difficulties, specific hypotheses have been formulated, mainly concerning the meta-cognitive level (Mariotti and Maffei, 2006). We suppose that the feedback offered by the microworld determines a change in the attitude towards errors and impasse. Zan (2002) gives a good starting point to investigate the relationship student - error mediated by the microworld:

> *"Even if the teacher recognises the student's error and intervenes, it is up to the student to modify his behaviour: but if the student is to significantly change his behaviour he first has to be convinced that the change has to be made, that the existing behaviour leads to failure."* (Zan, 2002, p. 94)

Actually, it is not easy for the teacher to enter the student - error relationship and make the student aware of the specific difficulty encountered. We think it's only by passing through this first step of *self-awareness* that it is possible to reach the second and final step consisting of the reformulation of the correct concept involved. In this way students are stimulated to be aware not only that something is wrong, but also to be aware of *what* is wrong.

We now report a typical behaviour detected in the first test passed. It contains exercises based on the expansion of main products. Antonio has experienced difficulties in applying the formulas during the sessions in the *training mode*. He has shown how to overcome these difficulties using the command *independent step* that is, opening a new environment where he has developed two different strategies. Actually, ahead of the error signal, the student has either used the distributive law to solve the exercise (avoiding the direct application of the specific formula required) or tried to obtain the required formula. Both of the schemes described have been completely accomplished in the *independent step* and only the solution has been copied in the resolution sequence. Figure 2 shows a solution given by the student in the *Test mode* and observed after the end of the test.

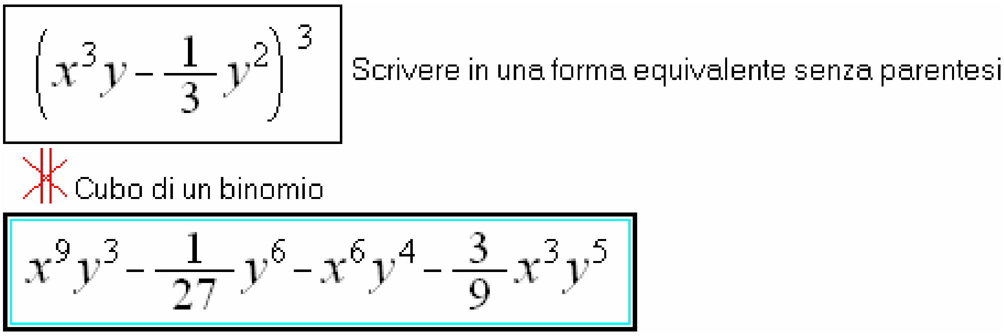

Figure 2 Antonio's solution during a test. The "non-equivalence" feedback was displayed later, when he revised his work (when he made the calculation, he got no feedback). The instruction means "Write in an equivalent form without parentheses" and the comment means "Cube of binomial".

The correct version of the exercise he gave in his notebook having looked at his work (accomplished in the *test mode*) in the *training mode* is shown in Figure 3. This experiment has been conducted with Aplusix 1.35 in which it was not possible to correct a test but only to observe the work. So, we asked pupils to correct their errors on paper. The student is seen to have identified the type of error; actually he wrote "*Maybe the signs?*". This hypothesis suggested to him a more global approach, so he created a *independent step* where he decided to get the formula of a cube of a difference, as if he was in the *independent step* phase of Aplusix. In this sense, we can see how much the student has internalised the control offered by the microworld. In fact, when he wrote on paper, he used a structured approach (that is, respecting the equivalence between two following lines) similar to that used when he worked in Aplusix.

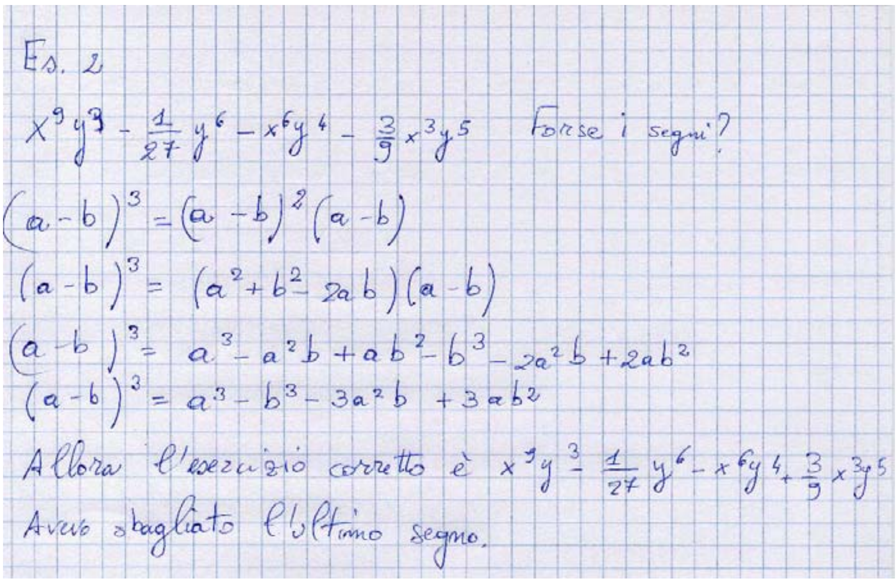

Figure 3 Antonio's self-correction on his notebook. The first comment means "Maybe the signs?". The final comment means "So the correct result is (…) I had got the last sign wrong."

Antonio, like many other students, seems to have achieved a good level of self-consciousness and self-control as a consequence of his work within Aplusix. Generally speaking, the control offered by the microworld seems to lead students to change the way they regard the error. We report an extract from an interview with a student involved in the experiment:

> *"Aplusix has some good features: everything seems easier than in paper and pencil. When I see the red lines, I understand that I have made an error (or more than one), I like it very much.....when my teacher corrects my test I don't look at the errors, the most important thing to which I pay attention is the good or bad mark I got."* (Ylenia)

This comment shows a development in perceiving one's own errors. In particular, it is interesting to remark how the student compares the feedback provided by Aplusix with the teacher feedback on errors. It is significant to notice how it is completely in accordance with the words of Zan reported above who argues that the students, first of all, have to clearly realise their errors, before they make every effort to overcome the encountered difficulties.

Conclusion

In spite of its limitations, the pilot experiment has provided good evidence confirming our hypotheses. All the students involved in the experiment have benefited from the use of the microworld, by improving their performance in algebraic calculation, as well as by improving their performances at the meta-cognitive level.

Our present research interest is to carry out a more detailed analysis concerning both the cognitive and the meta-cognitive processes involved in the use of Aplusix.

3 REMEDIATION IN USUAL SITUATION IN BRAZIL

In March and April 2004, as part of the Capes-Cofecub programme between Brazil and France, we organised a large test in the city of Campo Grande, Brazil. This test involved 2400 students of grade 9 and used the Aplusix system. The subject was linear equations and inequalities. Our objective was to gather data for studying the students' difficulties in this area of algebra. We observed that the success rate was very weak, although the topic has been introduced in grade 7. As a consequence, we decided to conduct a remedial experiment on equations with two grade 9 classes and a teacher who was interested in the project.

Methodology

Aplusix has been chosen for this experiment because the exercises of the map of exercises and the feedbacks of the system seemed to be adequate to provide to the expected remediation. The task of the teacher is facilitated by the map of exercises. The teacher can choose the families of exercises for training and testing the students. One important idea in this experiment was to let the teacher organise her work in order to see if such use of Aplusix was easy. The protocol adopted was as follows: each student solves the exercises within a family, beginning with the first family of equations, in the *training mode*, and passes a test with this family before going to the next one. Table 1 shows typical exercises from each family.

Family	An exercise	Another exercise
1	$6x = 30$	$-5x = 9$
2	$\frac{1}{2} = \frac{1}{3}x$	$\frac{5x}{3} = 2$
3	$-x - 7x - 1 = -8x - 4x + 1$	$-(x - 8) = 2(7x - 7)$
4	$\frac{3}{4}x + \frac{1}{8} = -\frac{1}{8}x - \frac{7}{16}$	$10(0,2x + 4) = 6$
5	$\frac{5}{2}(3x - 8) = \frac{9}{2}x + \frac{5}{2}(-4x + 9)$	$-4(-8x + 8) - 3(-7x - 4) + 5 = -7(6x - 8) - 5(6x - 7)$
6	$\sqrt{3}x + 5x = \sqrt{3} - 4x$	$\sqrt{4}(-2x + 6) = -2(\sqrt{4}x + 7)$

Table 1 Typical exercises from the different families.

The goal was to individualise the work by providing autonomy to the students who were allowed to go to the next family when they were satisfied by the score of the test. During training, the students get immediate feedback from Aplusix to help them overcome their difficulties. At the end of a test, they have an evaluation of their work (a score) and they can try to understand their errors in the *self-correction mode*. The teacher has the option of helping the students individually, going from one to another who raises his/her hand, or of providing explanations to all the students when she considers that almost all the students encounter the same difficulty. The work was conducted during the normal schedule of the class, in parallel with the teaching of the curriculum, during the last four months of the school year, with one session of one or two hours each two weeks, depending of the availability of the computer laboratory. The sessions were under the leadership of the teacher, the researcher only participated in the first session, to help during the familiarisation phase, and in the last one, to conduct a test. A table was to be filled by the teacher indicating the date, duration of each session and the family of exercises reached by each student at the end of the session.

Realisation and analysis of the experiment

The level of autonomy of the students increased from session to session. Each student worked according to his/her level which is very different from the usual classroom work on paper where all students have the same task. In the usual work, the teacher gives a list of selected exercises to all the students. Some students find the exercises too easy and are annoyed; others find the exercises difficult and are stuck. They would benefit from other situations, with easier exercises for example. The teacher does her best to help the student but has little time for each. With Aplusix, the students progressed according to their own pace so they were using different families of exercises during the same session. An important observation is that most of the students of both classes wanted to achieve a good score in the tests. Each time the score did not correspond to their expectations, they reworked the test until they achieved a good score. Usually, when they work on paper, the students are happy to have a score of 14 out of 20. With Aplusix, they always wanted to have more than 18 out of 20. As the teacher did not give directives on the scores, this new issue was created entirely by the students. So, trying to get a good grade led these students to come to grips with their learning. Each time they made errors in an exercise, they tried to see what was wrong.

Not all the students were present at all the sessions. Of the students, 54 participated in at least 5 sessions. As they chose the moment they changed to another family of exercises, their progressions were different. Table 2 shows these progressions. The majority of the students (34) reached families 3 or 4. With a traditional teaching style the students would have been confined to these families whereas the Aplusix environment allowed 19 students to go beyond and 3 to stay below.

Number of hours the students worked	5			6				7					8					9			
Family reached	3	4	5	2	3	4	5	2	3	4	5	6	2	3	4	5	6	3	4	5	6
Number of students who reached this family	1	1	1	1	1	3	5	1	3	2	1	2	1	5	2	5	1	8	6	2	2

Table 2 Distribution of the students in relation to the time spent and the families of exercises reached. Note that 21 students did not go beyond families 2 or 3 and that 19 students reached families 5 or 6. Note also that the majority of the students (34) reached families 3 or 4, and that there is no significant correlation between the time spent and the family reached.

The role of the teacher in this experiment was mainly limited to offer hints at specific moments. During the evaluation of the experiment, she said that, unlike the usual work in class, the work in the laboratory gave her more time to help students having difficulties, which allowed a better development for these students. Another remark made by the teacher concerns the contribution of Aplusix to her daily work: the map of exercises allowed her to suggest different exercises to students, better suited to their ability, without having to write them herself.

At the end of this experiment, the students were tested and the results analysed. Compared to the first test (March and April 2004), the results show an important development of the individual students. However, some remaining difficulties were observed which indicates that there were not enough sessions for having each student engaged with all the families of linear equations exercises.

An important methodological point of this experiment is the autonomy of the teacher: there was no direct intervention by the researcher. The decisions were made by the teacher, there were no prepared lists of exercises and there was no indication concerning the pace.

The results of this experiment, as well as the level of the usage of the Aplusix system and the level of learning of the students, led us to plan a new experiment with a similar protocol (no intervention by the researcher). A group of 8 teachers will integrate Aplusix in their teaching practice, either for remediation or for introduction of a concept. These experiments will take place in 2 classes of grade 7, 8 classes of grade 8, and 8 classes of grade 9. The students will work with Aplusix every 15 days. The data will be used to evaluate the students' difficulties in the learning of algebra, the role that Aplusix can have in this learning, and the way the teachers integrate Aplusix in their work.

4 COLLABORATIVE LEARNING IN INDIA

Can groups of children working together with technology, improve their Algebra scores, with the Aplusix software? This question inspired the study of Aplusix use by learners in an Indian school. Earlier work in 'Minimally Invasive Education' describes the process and the results of groups of children working together at computers without adult intervention (Mitra and Rana, 2001 and Inamdar, 2004). The work shows that, given adequate access, groups of children are able to teach themselves computing skills on their own without adult intervention. We seek to examine if Aplusix, used by groups of children, improves the individual's Algebra scores on tests. A study of Aplusix use and impact was conducted with 88 Grade 8 learners from a girl's school in the city of Mumbai (Bombay) for one-month in 2005. All learners participating in the study were taught the algebraic topic of 'Expansion' in school in 2004. There was no curricular teaching of this topic during the experimental period. All learners were computer literate and were randomly assigned to workgroups of about five to one computer.

Methodology

Each of twenty one workgroups was exposed to four 1-hour sessions of Aplusix, over one month, in the school's computer laboratory, starting with a ten-minute demonstration of how to use the program. The students used the Aplusix domains of 'Numerical Calculation' and 'Expansion' during the period of study. There was no classroom teaching of this during the experimental period. Hence we assume that any gains seen on post-test results over pre-test results could not be attributable to classroom teaching of the domains.

Two pre-tests and two post-tests were conducted for each student. One pre-test and post-test was extracted from the Aplusix program and conducted on paper. This gave us the Aplusix evaluation of learner performance. The second pre-test and post-test was created and administered by the teachers of the school, on paper, based on norms and standards of the Grade 8 curriculum in the school. This gave us the school's evaluation of learner performance. The reason for conducting two types of tests was to have an independent evaluation of the children's performance, from the school's perspective, before and after the Aplusix exposure.

Observations and Usage Statistics

Animated problem solving was observed within workgroups, followed by claps of joy when software feedback was received during Aplusix sessions. A usage tracking functionality in the Aplusix program tracked usage by all groups. Table 3 below shows some of this data in the domain of 'Expansion'. It shows that 'Grade 8' groups, spent an average of 124 minutes with 'Expansion' and the average time spent on each exercise was 2 minutes. The average number of attempted exercises per list of 10 exercises was 7 and the average number of well-solved exercises per list was 6. It appears from this data that the rate of problem solving in groups was high at one exercise in every 2 minutes. To examine this, in Table 3, we also attempt a comparison with French school students who worked individually on computers.

Names of classes	Avg. age of learners	Number of learners	Total time spent on expansion (in minutes)	Avg. exposure per learner (in minutes)	Avg. time per exercise	Avg. attempted exercises per list	Avg. well solved exercises per list	Avg. scores per list
Grade 8 (India)	13	21 groups	2607	124	2	7	6	16
Seconde 5 (France)	15	31	1285	42	2	7	6	16
Seconde 9 (France)	15	32	749	23	2	4	3	12
Troisieme (France)	14	15	470	31	3	4	1	8

Table 3 Usage Statistics from Aplusix

Note that:

1. The average exposure to the Expansion domain is more for the Indian students at 124 minutes compared to 42 minutes for the French 'Seconde 5' class, 23 minutes for the French 'Seconde 9' class and 31 minutes for the 'Troisieme' class.
2. The average time per exercise, the average attempted exercises per list, the average well solved exercises per list and the average scores per list are exactly the same for the 'Grade 8' Indian students aged 13 and the 'Seconde 5' French students aged 15.
3. The Indian students aged 13 are ahead of 'Troisieme' and 'Seconde 9' students aged 14 and 15 on the average attempted exercises per list, the average well solved exercises per list and the average scores per list.

Causes for the exact similarity in average usage statistics between 'Seconde 5' and 'Grade 8' despite the age difference could be many and can only be conjectured at this stage, as follows:

- Amount of exposure to Aplusix – the Indian students spent more time on Expansion than the French students. Could this increased exposure have significantly impacted the rate and success of problem solving?
- Impact of collaborative work – the Indian students worked in groups. Could the collaborative effort have significantly impacted the rate and success of problem solving? A number of qualitative comments from learners at the end of the study point to this. A few of their comments are presented here – "We loved the group working. It was then easier to solve the difficult sums of math". "I like to work in a group. We enjoyed very much. First time we were able to do all sums without anyone's help. We can increase our concentration".
- Prior knowledge– did the 13 year olds in India come to Aplusix with higher domain knowledge of Expansion than the 15 year olds in France? It appears not, as later results show that 89% of the Grade 8 class in India scored only 0-2 points, out of a total of 10, in the Aplusix pre-test for Expansion. Hence it is unlikely that prior knowledge has impacted the usage data in India.

A deeper pedagogical study of Aplusix usage in a collaborative context is required to find out the cause for this similarity in students that are 2 years apart in age and in two different continents.

Pre and Post test Results

Pre-test versus post-test comparisons in the 'Expansion' domain show that the average score went up by 33% on the school post-test and by 50% on the Aplusix post-test for the Grade 8 learners. More interesting for the schoolteachers, was the movement seen in the percentage of students in three score ranges before and after as shown in Table 4.

	Average group score	% Students in 0-2 score range	% Students in 3-7 score range	% Students in 8-10 score range
School pre-test *n*=76	3	45%	51%	5%
School post-test *n*=76	4	31%	48%	20%
Aplusix pre-test *n*=83	1	89%	11%	0%
Aplusix post-test *n*=83	2	64%	33%	3%

Table 4 Scores: the maximum score is 10.

Although the pattern of these shifts differs between the school test and the Aplusix test, it is clear that students have moved up score ranges. According to the school test, 45% of the class was in the low score range (0-2), 51% in the mid-score range (3-7) and 5% in the high score range before the Aplusix experience. This changed to 31% in the low scores, 48% in the mid-scores and 20% in the high scores at the end of the study.

According to the Aplusix pre-test for Expansion, 89% were in the low score range, 11% in the mid-range and none in the high score range. This changed to 64% in the low range, 33% in the mid range and 3% in the high score range at the end of the study.

Learner comments

At the end of this study, 78 students described their experiences in response to an open ended question. The students were asked to write about what they liked and did not like about the Aplusix learning experience. All 78 commented that they liked the program with responses being positive. The words "like", "love", "enjoy" and "fun" appear 135 times through the 78 comments. Students commented that Aplusix had helped them with improving their speed; it gave them more practice than they got from textbooks and class, it increased their "knowledge", it helped in solving "big sums", it improved their "mental math" and it helped them when they made a wrong step. Eight students commented on liking the ability to test oneself. There were 7 students who pointed to their difficulty with exercises involving square roots. Of all responses, thirteen made specific reference to and comments on their positive experiences with, the collaborative environment. Some evocative comments are given below:

"...We loved the group working. It was then easier to solve the difficult sums of math. We were given the chance to judge ourselves in math..."
"I like to work in a group. We enjoyed very much. First time we were able to do all sums without anyone's help. We can increase our concentration"
"...I have first time in my life worked on math in the computer. This program helps when we are wrong in our steps. This is what impressed me a lot...."
"I liked the program Aplusix because we get our mind developing in certain ways while using it. We can improve our mathematics..."
"...It also helped us to solve hard sums in math. In exam we used to do very small amount of sums in 1 hour but in Aplusix we did more than 40 sums in one hour"
"I liked this program because it helped me a lot in math and also gave us a different confidence of doing the toughest sum on our own"
"....My speed has also increased and it is simply fantastic! It even helped me to improve my calculations"

Conclusion

These results indicate that: (1) Group work with Aplusix has improved the performance of students as judged both by the school system as well as tests generated by the software program, although the difficulty level of the two tests appears to be different. (2) Usage statistics show that productivity of students of the Grade 8 class working in groups appears to be high. In this context, further research is needed to assess the pedagogical impact of collaborative work versus individual work with Aplusix as well as the impact of exposure time.

4 REGULAR USE IN FRANCE

The interest in the use of software in the teaching of mathematics has been demonstrated (Noss and Hoyles (1996)), however a not insignificant resistance to its use has been observed in France (Artigue, 1998). This resistance is not only due to a lack of knowledge of the tool by the teachers, but also to factors linked to the management of the class and to the design of teaching situations.

We have observed the use of Aplusix by two mathematics teachers at grade 10. The goal was to determine the way they integrate Aplusix into their teaching during the whole school year. This study was carried out in collaboration with Institut National de Recherche Pédagogique (INRP). At the end of the first year, we obtained a balance sheet of their use of the software. During the second year, we asked the teachers to complete forms with the following information: initiation, use of the various functionalities, duration, moment of use, type of activity, contents and goals, and impact on the students. Here are the results.

- **Initiation.** The students have no difficulty in familiarising themselves with the system and to use the basic functionalities. This is due to the good quality of the interface.

- **Use of the functionalities.** At the beginning, the students did not use the editing facilities; after the duplication of the expressions, they delete the step and input a new expression. After two or three sessions, and following suggestions from the teacher, the students began to use the editor, especially the drag and drop function. The verification of the calculations is correctly used by the students from the beginning.

- **Duration, moment of use, type of activity.** Teacher JMG used Aplusix during 6 sessions per student while teacher SM used it during 12 (note that the computer laboratory can only accommodate half the class at a time). At grade 10, the teaching is organised into three types of activities: *lesson and exercises* with the entire class; *reinforcement* with half a class; and *individualised-help* with students having difficulties. JMG and SM used Aplusix during *reinforcement* and *individualised-help* sessions according to the following sequence: *lesson and exercises* without Aplusix → *reinforcement* with Aplusix → *lesson and exercises* without Aplusix → *individualised-help* with Aplusix. In addition, some students in SM's class having difficulties used Aplusix at home or during free sessions of the computer laboratory. The students used the map of exercises for all the sessions. During *reinforcement*, the family of exercises was identical for all the students (chosen by the teacher); during *individualised-help*, each student used the most appropriate family of exercises for his/her knowledge level.

- **Contents and goals, impact on the students.** Aplusix has been used in each algebra topic of the grade 10 curriculum: factorisations, expansions and reductions, linear equations and inequalities, quadratic equations solved by factorisation, and systems of two linear equations in two unknowns. The teachers noticed that the students are autonomous during sessions with Aplusix. As a consequence, they can spend most of their time helping those who have great difficulties with algebra. In September of the second year, the ratio of SM's students having a score higher than 10 out of 20 was 25% and by March, it was 68%. Globally, the teachers noticed a significant improvement of the students' results with regard to the previous years (when they didn't use Aplusix).

Example 1: Sequence on equations at grade 10

We describe a use of the Aplusix system in SM's class during the school year 2002-2003 within the algebraic part of the curriculum. In September 2002, before any teaching of algebra, we gave a pre-test to the students. This test was made within the Aplusix environment and contained exercises of the different algebraic topics of grade 9. The analysis of the students' results revealed that there still existed difficulties in applying the knowledge learned in grade 9. As a consequence, we prepared a teaching sequence on two topics: factorisation and equation solving. These sequences were lists of exercises that students had to solve in the *training mode* of Aplusix. After that a post-test was administered for comparison with the pre-test. The percentage of well-solved exercises increased from 18% at pre-test to 68% at post-test. This evolution depended on the type of the exercise, and also, on subcategories of problems. For example, about type of exercise "Solve $ax+b=cx+d$", the success for exercises with integer coefficients was 100% in the post-test whereas it was only 30% for exercises with non integer coefficients. As students only worked with Aplusix during this period, we make the hypothesis that the evolution of the results is due to the *milieu* provided by Aplusix and to the choice of the situations.

Example 2: Sequence on systems of equations at grade 10

Usually, students do not solve systems of linear equations by equivalence, writing successions of equivalent systems like they have to do with Aplusix. When they use the substitution method, they generally work first with one equation, expressing one variable in terms of the other(s) and then substitute in another equation. When they use linear combinations, they produce new equations, one after the other.

Three grade 10 classes used Aplusix, in the *training mode*, during two one-hour sessions for learning how to solve linear equations (two equations with two unknowns). At the beginning, they didn't produce equivalent systems at each step, sometimes writing a unique equation (see Figure 4 with linear combinations). But the *milieu* forced them to work with equivalence because it told students that there was no equivalence (see Figure 4).

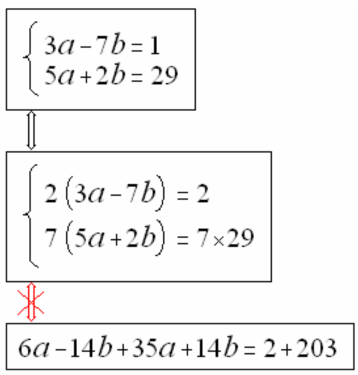

Figure 4 The student prepares a linear combination in the second step by multiplying each equation by an appropriate number. Then he adds both equations and provides the result in

a third step. Aplusix indicates that step three is not equivalent to step two.

Later, the students had to solve systems of equations on paper. Most of them (90%) produced equivalent systems on paper, whatever method they used.

APLUSIX as an environment for doing experiments.

The reinforcement phase used the training mode with a verification of the equivalence on request (the students had to click on a button when they wanted to display the equivalence, and to use the resulting information). This introduces Aplusix as an environment for doing experiments. While performing actions and observing feedback, students developed their abilities, their controls, their strategies, and were able to correct their errors (see Figure 5).

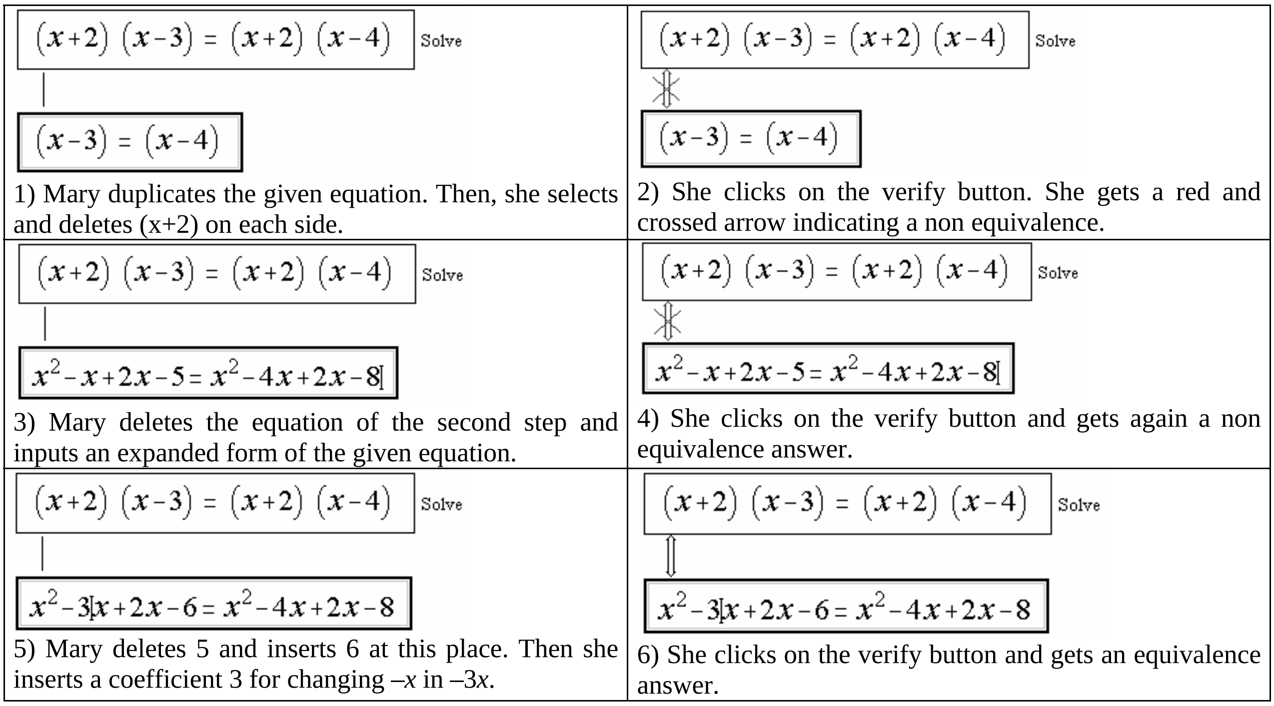

Figure 5 Beginning of the resolution by Mary of the exercise "Solve the equation $(x+2)(x-3)=(x+2)(x-4)$". The equivalence between two steps is indicated to Mary when she clicks on the verify button

Conclusion

The cost of integrating Aplusix into the mathematics class is low and its use is very efficient. Globally, the teacher observed that the students changed their answers more easily with Aplusix than on paper, and did not hesitate to test new strategies. The teachers also highlight the fact that the students can work autonomously. Furthermore the map feature that provides families of exercises with coefficients randomly assigned, reduces their preparation workload.

6 GENERAL CONCLUSION

Through many experiments, in different countries and contexts, Aplusix has been shown to be a *usable* computer program: it is easy for the student to use and is very "user friendly". It is *useful*, favouring the students' learning of the algebra curriculum, as has been proven by pre- and post-tests on paper. Through its efficient feedbacks and the provision of exercises with random coefficients, Aplusix *lightens* the teachers' task, so teachers can concentrate their efforts on helping students who have great difficulties with algebra. This feature must be underlined because it is very rare among computer programs for education. Aplusix is also a tool for researchers in mathematics education, allowing the off-line study of students' work. These characteristics will encourage the development team to reinforce its efforts for promoting the widespread use of Aplusix in the world.

7 REFERENCES

Artigue M. (1998) Teacher training as a key issue for the integration of computer technologies, D. Tinsley and D. C. Johnson (eds), *Information and communications technologies in school mathematics*, London: Chapman and Hall, 121-129.

Freudenthal, H. (1983) *Didactical phenomenology of mathematical structures*, Dordrecht: Kluwer Academic Publishers.

Inamdar, P. (2004) Computer skills development by children using 'hole in the wall' facilities in rural India. *Australasian Journal of Educational Technology*, **20**, 337-350.

Kieran, C. (1992) The learning and teaching of School Algebra, in Grows, D. A. (ed) *Handbook of Research on Mathematics Teaching and Learning,*, New York: NCTM.

Mariotti, M.A. and Maffei, L. (2006) Difficoltà in algebra: un intervento di recupero, *La matematica e la sua didattica*, **1**, 81-99.

Mitra, S. and Rana, V. (2001) Children and the Internet: Experiments with minimally invasive education in India. *The British Journal of Educational Technology*,**32**, 221-232.

Nicaud, J.F., Bouhineau, D. and Chaachoua H. (2004) Mixing Microworld and CAS Features for Building Computer Systems that Help Students to Learn Algebra. *International Journal of Computers for Mathematical Learning*. **9**, 169-221.

Norman, D. (1988) *The psychology of everyday things*, New York: Basic Book.

Noss R and Hoyles (1996) *Windows on Mathematical Meanings*, Dordrecht: Kluwer Academic Publisher.

Tall, D. and Thomas, M. (1991) Encouraging versatile thinking in algebra using the computer, *Educational Studies in Mathematics*, **22**, 125-147.

Zan, R. (2002) Episode II: Marco and Anna - Each following their own path da Learning from learners, Cockburn, A. D. and Nardi, E. (eds), *Proceedings of the 26th PME Conference*, Norwich: University of East Anglia.

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Marilena Bittar is an associate professor in mathematics education of the Universidad Federal de Mato Grosso do Sul (Campo Grande, Brazil). She is the head of a research team in mathematics education working on the learning of algebra. She currently studies the learning of algebra with ICT and working on students' modelling in algebra.

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Parimala Inamdar heads the Design Laboratory at the Centre for Research in Cognitive Systems, NIIT, India. She has worked in the area of learning technologies since India's pioneering interactive videodisc project in 1989. Her research interests include cognition, collaboration and the design of technology enhanced learning environments for children.

Laura Maffei took her degree in 2004 in Pisa (Italy) under the guidance of Prof. M.A. Mariotti. Her thesis considers the role that the Aplusix learning environment can play in an intervention devoted to students (9^{th} grade - first year of the Italian High School) having some difficulties in the learning of Algebra. Her current research interests concern the impact of the use of Aplusix at the cognitive and metacognitive levels.